Relativistic velocity addition and the relativity of space and time intervals

J.H.Field
Département de Physique Nucléaire et Corpusculaire, Université de Genève
24, quai Ernest-Ansermet CH-1211Genève 4.
E-mail: john.field@cern.ch

Abstract

A thought experiment first proposed by Sartori is analysed using the parallel velocity addition formula of special relativity. The spatial and proper-time intervals between some similarly defined spatial coincidence events are found to be widely different in different inertial frames. This relativity of space and time intervals is quite distinct from the well-known time-dilatation and length contraction effects of special relativity. Sartori's claimed derivation of the parallel velocity addition formula, assuming relativistic time dilatation, based on the thought experiment, is shown to be fortuitous.

PACS 03.30.+p

This paper analyses a thought experiment, that was orginally proposed by Sartori [1], involving two trains T1 and T2, moving at different speeds v and u respectively relative to a fixed platform P. It was claimed that the parallel velocity addition relation (PVAR) could be derived from the experiment by assuming the existence of the relativistic time dilatation effect [2]. The derivation will be critically examined below; however, the main purpose of the present paper is a different one. *Assuming* the PVAR, space and time intervals between spatial coincidences, in the direction of motion, of P, T1 and T2 are calculated in their respective rest frames S, S' and S" and compared. .

In Fig.1 is shown the experiment as preceived by an observer in the rest frame of P. It is assumed that there is an array of synchronised clocks in each frame, each of which indicates a common 'frame time' (i.e. the proper time of any of the clocks) τ, τ' and τ'' in the frames S, S' and S". At frame time $\tau 1 = 0$ the train T1 is opposite the platform (Event1) and T2 is at a distance $L1$ from it (Fig1a). At frame time $\tau 2$, T2 is opposite the platform (Event2) and T1 is at a distance $L2$ from it (Fig1b). Finally, since it is assumed that $u > v$, at frame time $\tau 3$, T1 and T2 are opposite each other, at a distance $L3$ from P (Event3). Figs.2 and 3 show the same sequence of events as observed in the rest frames of T1 and T2 respectively. The corresponding frame times and distances are $\tau 1'$, $\tau 2'$, $\tau 3'$; $L1'$, $L2'$ $L3'$ in the rest frame, S', of T1 and $\tau 1''$, $\tau 2''$, $\tau 3''$; $L1''$, $L2''$ $L3''$ in the rest frame, S", of T2. In Fig.2, T2 and in Fig.3, T1 move at the speed w given by the PVAR [2]:

$$w = \frac{u - v}{1 - \frac{uv}{c^2}} \tag{1}$$

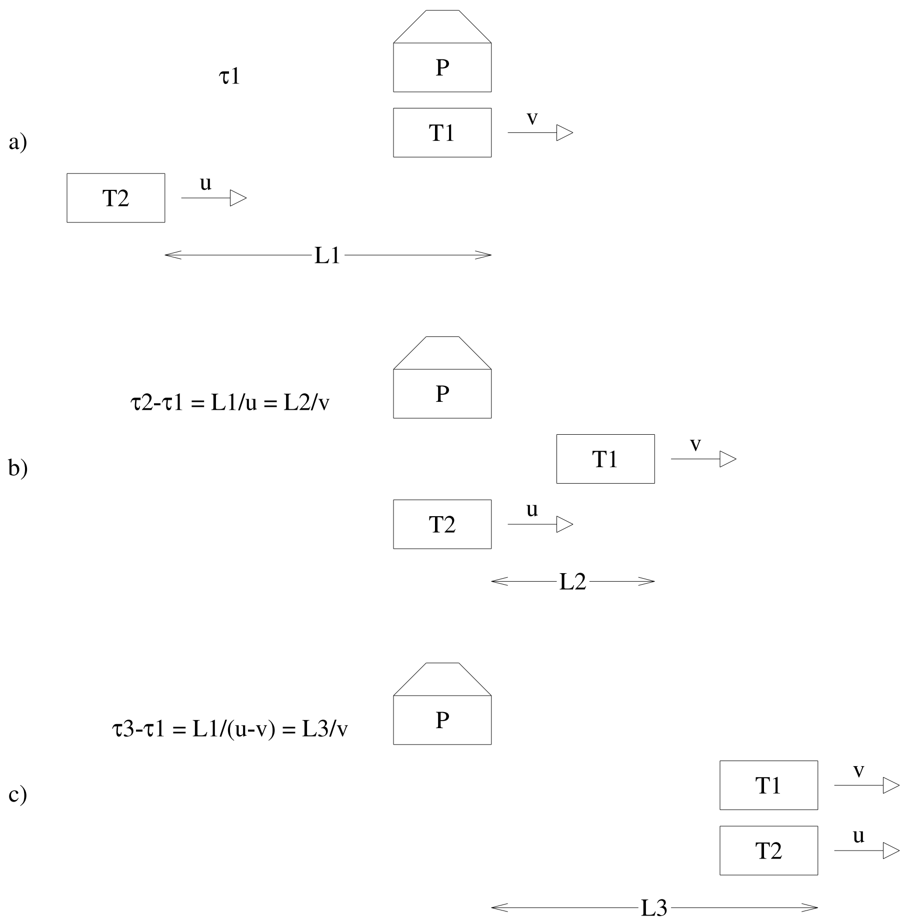

Figure 1: *Spatial coincidence events as observed in the rest frame, S, of P. a) Event1, T1 opposite P, b) Event2, T2 opposite P, c) Event3, T1 opposite T2.* $u = 0.8c$, $v = 0.4c$

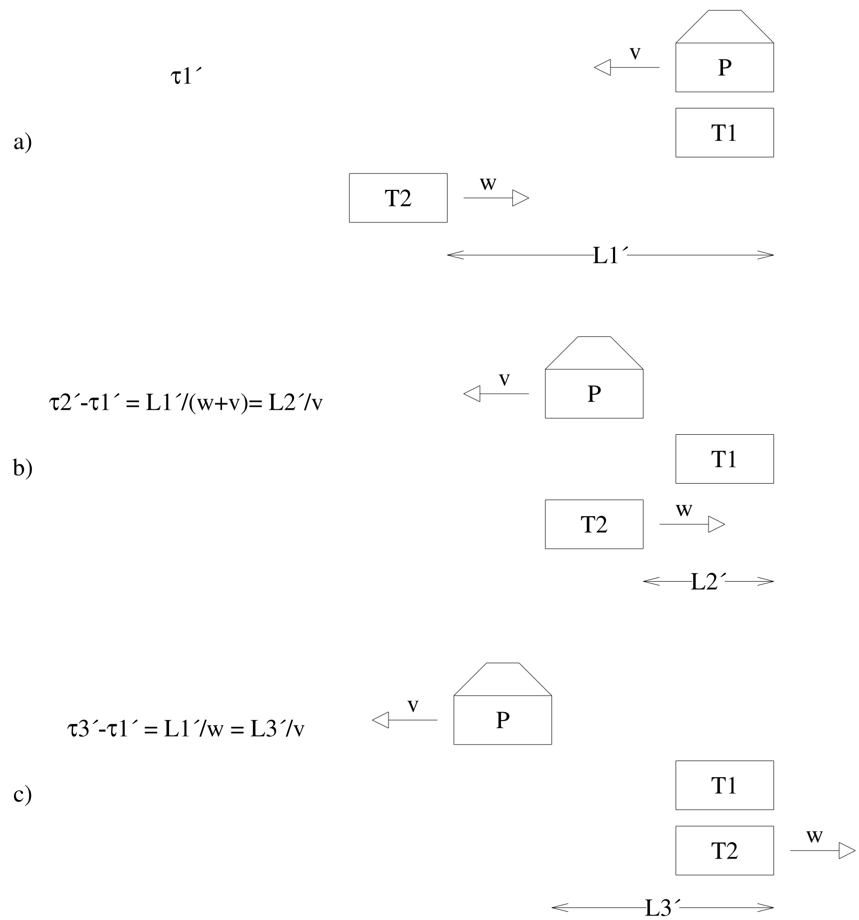

Figure 2: *Spatial coincidence events as observed in the rest frame, S', of T1. a) Event1, T1 opposite P, b) Event2, T2 opposite P, c) Event3, T1 opposite T2.* $w = 0.588c$, $v = 0.4c$

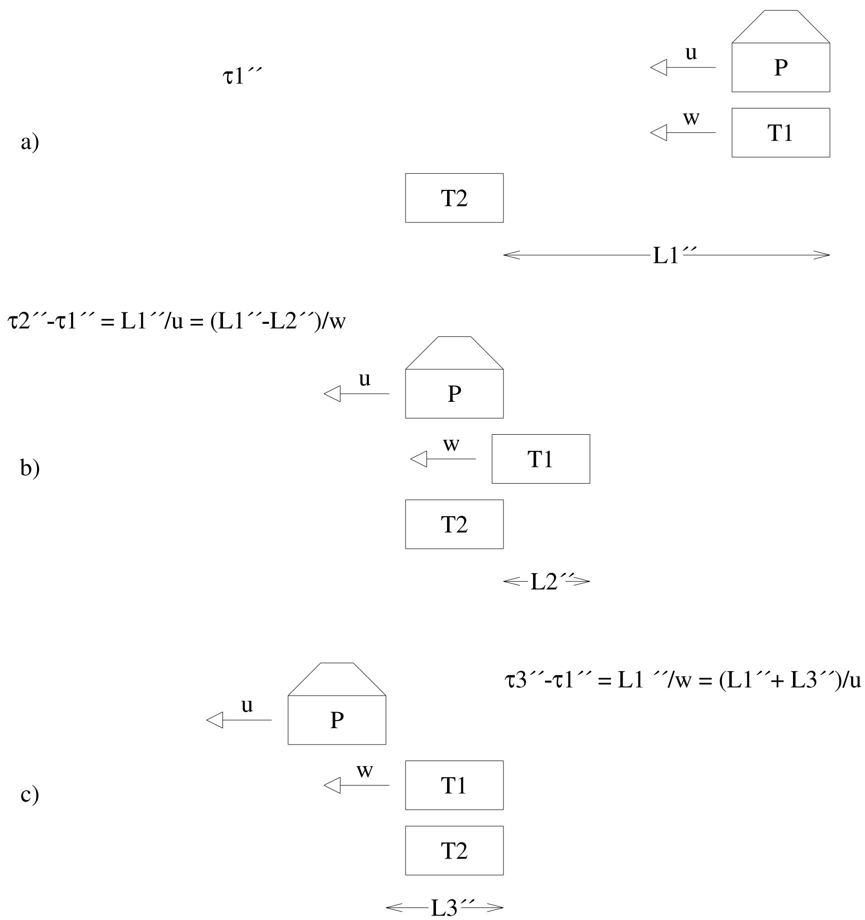

Figure 3: *Spatial coincidence events as observed in the rest frame, S", of T2. a) Event1, T1 opposite P, b) Event2, T2 opposite P, c) Event3, T1 opposite T2.* $u = 0.8c$, $w = 0.588c$

The geometry of Figs.1-3 gives the relations:

$$
\begin{aligned}
\tau 2-\tau 1 &= \frac{L1}{u},\quad L2=\frac{vL1}{u},\quad \tau 3-\tau 1=\frac{L1}{u-v},\quad L3=v\frac{L1}{u-v} && (2)\\
\tau 2'-\tau 1' &= \frac{L1'}{w+v},\quad L2'=\frac{vL1'}{w+v},\quad \tau 3'-\tau 1'=\frac{L1'}{w},\quad L3'=v\frac{L1'}{w} && (3)\\
\tau 2''-\tau 1'' &= \frac{L1''}{u},\quad L2''=\frac{u-w}{u}L1'',\quad \tau 3''-\tau 1''=\frac{L1''}{w},\quad L3''=\frac{u-w}{w}L1'' && (4)
\end{aligned}
$$

Eliminating w from (3) and (4) by use of (1), Eqns(2)-(4) yield for the frame times and spatial positions of Event2 and Event3 in the frames S, S' and S", respectively:

$$
\begin{aligned}
\tau 2-\tau 1 &= \frac{L1}{u},\quad L2=\frac{vL1}{u} && (5)\\
\tau 3-\tau 1 &= \frac{L1}{u-v},\quad L3=\frac{vL1}{u-v} && (6)\\
\tau 2'-\tau 1' &= \frac{1}{u}\frac{(1-\frac{uv}{c^2})}{1-\frac{v^2}{c^2}}L1',\quad L2'=\frac{v}{u}\frac{(1-\frac{uv}{c^2})}{1-\frac{v^2}{c^2}}L1' && (7)\\
\tau 3'-\tau 1' &= \frac{(1-\frac{uv}{c^2})}{u-v}L1',\quad L3'=v\frac{(1-\frac{uv}{c^2})}{u-v}L1' && (8)\\
\tau 2''-\tau 1'' &= \frac{L1''}{u},\quad L2''=\frac{v}{u}\frac{(1-\frac{u^2}{c^2})}{1-\frac{uv}{c^2}}L1'' && (9)\\
\tau 3''-\tau 1'' &= \frac{(1-\frac{uv}{c^2})}{u-v}L1'',\quad L3''=v\frac{(1-\frac{u^2}{c^2})}{u-v}L1'' && (10)
\end{aligned}
$$

In the above formulae the ratios of length and time intervals in each frame are fixed by the relative velocities u and v, but the lengths $L1$, $L1'$ and $L1''$ are arbitary. In order to find the the relation between these lengths it is necessary to use the condition that the events corresponding to the spatial coincidences T1-P, T2-P and T1-T2 are each observed simultaneously in the corresponding inertial frames[a].

Inspection on Figs.1-3 shows that there are nine distinct observations of spatial coincidence events in the problem: T1-P, T2-P and T1-T2 as observed in the frames S, S' and S". It is convenient to introduce the following notation to specify these different observations:

Event1	<u>P(S)-T1(S')</u>	<u>P(S)'-T1(S')'</u>	P(S)"-T1(S')"
Event2	<u>P(S)-T2(S")</u>	P(S)'-T2(S")'	<u>P(S)"-T2(S")"</u>
Event3	T1(S')-T2(S")	<u>T1(S')'-T2(S")'</u>	<u>T1(S')"-T2(S")"</u>

Here, for example, P(S) shows that the proper frame of P is S, and P is observed in S, whereas P(S)' indicates that P is observed in the frame S'. The other symbols are defined in a similar fashion. The underlined observations are *reciprocal* ones of the same spatial coincidence event. In each case, one object is at rest, and the other in motion with the same absolute velocity. These observations have the important property that

[a]These frames are S',S for T1-P, S",S for T2-P, and S',S" for T1-T2.

they are *mutually simultaneous*[b] in the proper frames of both objects. This property was used by Einstein [2] to define synchronous clocks in two inertial frames at the instant of spatial coincidence of the origins of the frames, by setting both frame times to zero at this instant. This procedure has been called 'system external synchronisation' by Mansouri and Sexl [3]. In order to mutually synchronise local clocks at the positions of P, T1 and T2, system external synchronisation may be applied at any two of the three pairs of reciprocal observations. For example P(S)-T1(S') and P(S)'-T1(S')' in S and S' respectively may be used to set $\tau 1 = \tau 1' = 0$. This was done in Sartori's analysis of the problem. The mutually simultaneous observations P(S)-T2(S") and P(S)"-T2(S")" may be then used to set $\tau 2'' = \tau 2$. However a clearer understanding of the problem is obtained by performing the analysis of the problem without introducing synchronised clocks. Such an analysis is now presented.

Since Event1 is mutually simultaneous in the frames S and S', it follows that a necessary condition that two other events with times τ and τ' are mutually simultaneous is that

$$\tau - \tau' = \tau 1 - \tau 1' \equiv \Delta\tau(\mathrm{S},\mathrm{S}') \qquad (11)$$

Similarly, since Event2 is mutually simultaneous in the frames S and S", two other events in these frames are mutually simultaneous provided that

$$\tau - \tau'' = \tau 2 - \tau 2'' \equiv \Delta\tau(\mathrm{S},\mathrm{S}'') \qquad (12)$$

It follows from (11) and (12) that events with times τ' and τ'' are mutually simultaneous in the frames S' and S", provided that:

$$\tau' - \tau'' = \Delta\tau(\mathrm{S},\mathrm{S}'') - \Delta\tau(\mathrm{S},\mathrm{S}') = \tau 2 - \tau 2'' - (\tau 1 - \tau 1') \qquad (13)$$

Since the Event3 *is* mutually simultaneous in the frames S' and S" it follows that:

$$\tau 3' - \tau 3'' = \tau 2 - \tau 2'' - (\tau 1 - \tau 1') \qquad (14)$$

This equation may be rearranged to give:

$$\tau 3' - \tau 1' = \tau 2 - \tau 1 + \tau 3'' - \tau 1'' - (\tau 2'' - \tau 1'') \qquad (15)$$

where $\tau 1''$ has been added to and subtracted from the right side. Combining (15) with Eqns(2),(3) and (4) gives

$$\frac{L1'}{w} = \frac{L1}{u} + \frac{L1''}{w} - \frac{L1''}{u} \qquad (16)$$

solving this equation for $L1''$:

$$L1'' = \frac{L1' - (w/u)L1}{1 - w/u} \qquad (17)$$

This equation holds for all values of w/u. Setting $w = 0$ then gives

$$L1'' = L1' \qquad (18)$$

[b] To be contrasted with simultaneous events in the *same* inertial frame, e.g. those specified by the same reading of two stationary synchronised clocks at different positions in the frame.

Using (18) to eliminate $L1''$ from (16) gives

$$L1 = L1' = L1'' \tag{19}$$

In view of (19) the following relations may be derived from (5)-(10):

$$\begin{aligned} \frac{\Delta\tau 21'}{\Delta\tau 21} &= \frac{1-\frac{uv}{c^2}}{1-\frac{v^2}{c^2}} = \frac{L2'}{L2}, \quad \frac{\Delta\tau 31'}{\Delta\tau 31} = 1-\frac{uv}{c^2} = \frac{L3'}{L3} && (20) \\ \frac{\Delta\tau 21''}{\Delta\tau 21} &= 1, \quad \frac{L2''}{L2} = \frac{1-\frac{u^2}{c^2}}{1-\frac{uv}{c^2}}, \quad \frac{\Delta\tau 31''}{\Delta\tau 31} = 1-\frac{uv}{c^2}, \quad \frac{L3''}{L3} = 1-\frac{u^2}{c^2} && (21) \end{aligned}$$

where, for example, $\Delta\tau 21 \equiv \tau 2 - \tau 1$. Setting $u = c$ in (20) and (21) gives:

$$\begin{aligned} \frac{\Delta\tau 21'}{\Delta\tau 21} &= \frac{1}{1+\frac{v}{c}} = \frac{L2'}{L2}, \quad \frac{\Delta\tau 31'}{\Delta\tau 31} = 1-\frac{v}{c} = \frac{L3'}{L3} && (22) \\ \frac{\Delta\tau 21''}{\Delta\tau 21} &= 1, \quad \frac{L2''}{L2} = \frac{L3''}{L3} = 0, \quad \frac{\Delta\tau 31''}{\Delta\tau 31} = 1-\frac{v}{c} && (23) \end{aligned}$$

The relativistic contraction of $L2'$ and $L2''$ relative to $L2$, and $L3'$ and $L3''$ relative to $L3$, is evident on comparing Fig.1 with Figs.2 and 3, where $u = 0.8c$, $v = 0.4c$ $w = .588c$. The reciprocal observations of the coincidence events are mutually simultaneous:

$$\frac{\Delta\tau 21''}{\Delta\tau 21} = \frac{\Delta\tau 31''}{\Delta\tau 31'} = 1$$

These relations exemplify the 'Measurement Reciprocity Postulate [4] that reciprocal measurements in two inertial frames yield identical results –in this case equal time intervals in the frames in which the measurements are performed

For the coincidences P(S)'-T2(S")' for Event2 and T1(S')-T2(S") for Event3, where both objects are in motion in the frame of observation, (20) and(21) give:

$$\frac{\Delta\tau 21''}{\Delta\tau 21'} = \frac{1-\frac{v^2}{c^2}}{1-\frac{uv}{c^2}} \neq 1 \quad \frac{\Delta\tau 31''}{\Delta\tau 31} = 1-\frac{uv}{c^2} \neq 1$$

so that that non-reciprocal observations of spatial coincidences (where, in one frame, both objects are in motion) are found to be not mutually simultaneous. Note that the relativity of lengths and times apparent in (20) and (21) is quite distinct from the time dilatation (TD) and length contraction effects of conventional special relativity. All times considered here are frame times recorded by stationary clocks in each frame, so that the Lorentz transformation, that relates a proper time recorded by stationary clock to the apparent time of uniformly moving one, plays no role, except insofar as the PVAR is a necessary consequence of the LT [2].

Sartori's derivation of the PVAR [1], on the basis of the thought experiment just discussed, is now considered. It is based on the relations, readily derived from Eqns(2) and (3) above:

$$\begin{aligned} \Delta\tau 31 &= \frac{u}{u-v}\Delta\tau 21 && (24) \\ \Delta\tau 31' &= \frac{w+v}{w}\Delta\tau 21' && (25) \end{aligned}$$

In order to derive the PVAR from these equations Sartori assumes that the frame time intervals $\Delta\tau 21$ and $\Delta\tau 21'$ and $\Delta\tau 31$ and $\Delta\tau 31'$ are connected by the TD relations [2]:

$$
\begin{aligned}
\Delta\tau 21' &= \gamma \Delta\tau 21 &\quad (26)\\
\Delta\tau 31 &= \gamma \Delta\tau 31' &\quad (27)
\end{aligned}
$$

where $\gamma \equiv \sqrt{1-(v/c)^2}$. The argument given by Sartori to justify (26) is that: 'Since Event1 and Event2 occur at the same place in S the interval $\tau 2 - \tau 1$' (in the notation of the present paper)'is a proper time interval'. For (27) the argument is: 'Similarly since Event1 and Event3 occur at the same place in S', the interval $\tau 3' - \tau 1'$ is a proper time interval'. These statements are correct, but since the times τ, τ', are those recorded by a synchronised clock at *any* position in the frames S, S' the fact that the time interval $\tau 2 - \tau 1$ is defined by spatial coincidences with P (P-T1 at the beginning of the interval and P-T2 at the end) has no special physical significance. Similarly, for the interval $\tau 3' - \tau 1'$, where the limits of the interval are defined by the spatial coincidences T1-P and T1-T2, the exact correspondence of this interval with the proper time interval of a clock at rest at the position of T1 is in no way different, given the possible existence of a synchronised clock indicating the frame time at any position in the frame, to a frame-time interval defined by spatial coincidences at different positions.

The experiment defined by the TD relation (26) is one in which a local clock situated at P is observed from the frame S', relative to which it is moving to the left with velocity v (Fig.2) If the frame-time interval $\Delta\tau 21'$ in Eqn(26) is that between the events P-T1 and P-T2 in S', it is not correct to substitute $\Delta\tau 21$, the frame time interval $\tau 2 - \tau 1$ in Fig.1 in the TD relation. In fact (26) and (27) should be written as

$$
\begin{aligned}
\Delta\tau 21' &= \gamma \Delta t21 &\quad (28)\\
\Delta\tau 31 &= \gamma \Delta t31' &\quad (29)
\end{aligned}
$$

where $\Delta t21 \equiv t_2 - t_1$ is the apparent time interval of a moving 'slowed down' clock at the position of P as viewed from S' and $\Delta t31' \equiv t'_3 - t'_1$ is the apparent time interval of a moving 'slowed down' clock at the position of T1 as viewed from S. In the thought experiment analysed above no such observations of moving clocks are performed, so the correct TD formulae (28) and (29) have no possible relevance to the problem. Indeed, Sartori's formulae (26) and (27) are incorrect. Eqns(20) and (21) give

$$
\begin{aligned}
\frac{\Delta\tau 21'}{\Delta\tau 21} &= \frac{1-\frac{uv}{c^2}}{1-\frac{v^2}{c^2}} \neq \gamma &\quad (30)\\
\frac{\Delta\tau 31}{\Delta\tau 31'} &= \frac{1}{1-\frac{uv}{c^2}} \neq \gamma &\quad (31)
\end{aligned}
$$

In contradiction to Eqns(26) and (27).

The hypothesis underlying (26) and (27) –the substitution of frame times defined by the coincidence events shown in Figs.1 and 2 into the TD relation– may be excluded on the grounds that it contradicts the assumed initial conditions of the thought experiment. Consider the experiment reciprocal to that described by the TD relation (28) i.e. a local clock at the position of T1 is viewed from the frame S. Although there is no local spatial coincidence to define the time interval $\tau 2' - \tau 1'$ in Fig.2, this interval is the same as that which would be recorded by a local clock at T1 at the instant of the P-T2 coincidence,

since $\tau 2'$ is the time of this coincidence as recorded by *all* synchronised clocks in S'. Sartori's ansatz then gives, for this reciprocal experiment, the relation

$$\Delta\tau 21 = \gamma \Delta\tau 21' \tag{32}$$

Similar consideration of reciprocal experiments, gives, with Sartori's hypothesis, the relation reciprocal to (27):

$$\Delta\tau 31' = \gamma \Delta\tau 31 \tag{33}$$

Combining (26) and (32) gives

$$\Delta\tau 21' = \gamma \Delta\tau 21 = \gamma^2 \Delta\tau 21' \tag{34}$$

It then follows that $\gamma^2 = 1$, $v = 0$, contradicting the initial hypothesis $v \neq 0$ of the thought experiment. The same conclusion follows on combining (27) and (33).

The apparent time intervals $\Delta t21$ and $\Delta t31'$ as observed in the frames S' and S (28) and (29) are also frame time intervals $\Delta t21 = \Delta\tilde{\tau}21 \equiv \tilde{\tau}2 - \tau 1$ and $\Delta t31' = \Delta\tilde{\tau}31' \equiv \tilde{\tau}3' - \tau 1'$ seen by observers in S and S' respectively. The spatial configurations in these frames given by (28) and (29) corresponding to the values of $\Delta\tau 21'$ and $\Delta\tau 31$ in Figs.2 and 1 respectively are shown in Figs.4 and 5. Combining (28) or (29) with (5)-(8) and (19) gives:

$$\begin{aligned} \Delta\tilde{\tau}21 &= \frac{1-\frac{uv}{c^2}}{\gamma u(1-\frac{v^2}{c^2})}L1, \quad \Delta\tau 21 = \frac{L1}{u} && (35) \\ \Delta\tilde{\tau}31' &= \frac{L1}{\gamma(u-v)}, \quad \Delta\tau 31' = \frac{L1}{w} && (36) \end{aligned}$$

With $u = 0.8c$, $v = 0.4c$, $w = 0.588c$ and $\gamma = 1.09$, as in the figures

$$\begin{aligned} \Delta\tilde{\tau}21 &= 0.93L1/c < \Delta\tau 21 = 1.25L1/c \\ \Delta\tilde{\tau}31' &= 2.3L1/c > \ \Delta\tau 31' = 1.7L1/c \end{aligned}$$

In Fig.4 $\tilde{\tau}2$ is less than $\tau 2$, whereas in Fig.5 $\tilde{\tau}3'$ is greater than $\tau 3'$.

Eqns(24) and (25) can be combined to give the relation

$$\frac{\Delta\tau 31}{\Delta\tau 31'}\frac{\Delta\tau 21'}{\Delta\tau 21}(1-\frac{v}{u})(1+\frac{v}{w}) = 1 \tag{37}$$

Substituting the ratios $\Delta\tau 31/\Delta\tau 31'$ and $\Delta\tau 21'/\Delta\tau 21$ from (20) gives

$$(1-\frac{v}{u})(1+\frac{v}{w}) = (1-\frac{v^2}{c^2}) \tag{38}$$

which, when solved for w in terms of u and v gives the PVAR (1). The same result is given by substituting the ratios $\Delta\tau 3/\Delta\tau 3'$ and $\Delta\tau 2'/\Delta\tau 2$ from Sartori's relations (26) and (27). Making the equally (in)valid substitution of frame times for apparent times to give the reciprocal TD relations of (32) and (33), and substituting these times into Eqn(37) gives

$$(1-\frac{v^2}{c^2})(1-\frac{v}{u})(1+\frac{v}{w}) = 1 \tag{39}$$

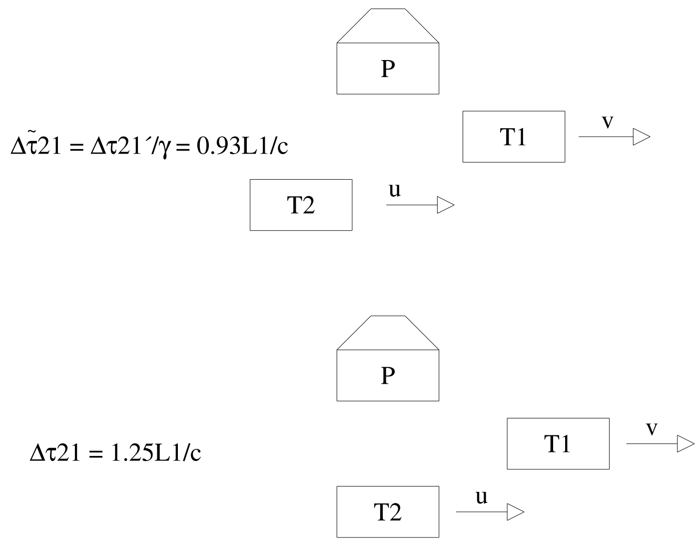

Figure 4: *Spatial coincidence events as observed in the rest frame, S, of P. Comparison of configurations at times $\tau 2$ and $\tilde{\tau} 2$. The event at time $\tilde{\tau} 2$ is related via the TD relation (28) with the event at time $\tau 2'$ in S'.*

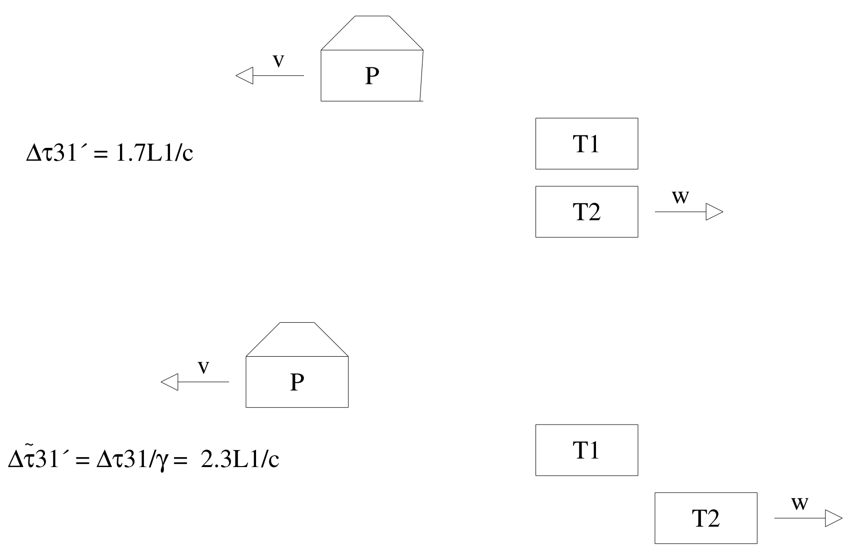

Figure 5: *Spatial coincidence events as observed in the rest frame, S', of T1. Comparison of configurations at times $\tau 3'$ and $\tilde{\tau} 3'$. The event at time $\tilde{\tau} 3'$ is related via the TD relation (29) with the event at time $\tau 3$ in S.*

which is derived by similar hypotheses to that used to derive (38), but does not yield the PVAR. In conclusion, Sartori's 'derivation' of the correct PVAR (1) on the basis of the the assumed (but incorrect) equations (26) and (27) is fortuitous.

Acknowledgement

I thank two anonymous refereees of another journal for critical comments that prompted me to improve the clarity of the presentation of two earlier versions of this paper.

Added Note

Some conclusions of the present paper are flawed by important errors both in calculation and of concept. The correct analysis of Sartori's thought experiment is given in a more recent paper by the present author [5]. The known calculational and conceptual mistakes that exist in the analysis of the present paper are explained in this Added Note.

The argument concerning mutually simultaneous events,. leading to Eqn(17), is logically correct. The aim of this calculation is to find the relation of $L1'$ and $L1''$ to $L1$, assuming the latter distance to be a fixed initial parameter of the problem, and that $L1'$ and $L1''$ are observed in the frames S', S" moving with speeds v, u, respectively, relative to the frame S in which $L1$ is specified. The separations $L1'$ and $L1''$ must then have (if any) a similar velocity dependence: $L1' = L(L1, v)$, $L1'' = L(L1, u)$ where $L1 \equiv L(L1, 0)$. Thus Eqn(17) should be written:

$$L(L1, u) = \frac{L(L1, v) - (w/u)L(L1, 0)}{1 - w/u} \tag{40}$$

If $w = 0$ then $u = v$ so that (40) does not give, as claimed, the relation $L1'' = L1'$ but instead the trivial identity:

$$L(L1, v) \equiv L(L1, v) \tag{41}$$

However Eqn(19): $L1 = L1' = L1''$, is correct. To show this, introduce x-coordinate axes in the frames S and S" parallel to the direction of motion of the trains, with origins aligned with the position of P in Fig.1a at $\tau = \tau1$. The world lines of T2 in S" and S respectively are then

$$x''(\mathrm{T2}) = -L1''(\text{all } \tau''), \quad x(\mathrm{T2}) = u(\tau - \tau1) - L1$$

corresponding to the space Lorentz transformation equation:

$$x''(\mathrm{T2}) + L1'' = \gamma_u[x(\mathrm{T2}) + L1 - u(\tau - \tau1)] = 0 \tag{42}$$

where $\gamma_u \equiv 1/\sqrt{1 - (u/c)^2}$. The interval $L1''$ is a fixed constant and $L1 \equiv -x(\mathrm{T2}, \tau = \tau1)$ is also a constant, which is independent of u, reflecting the choice of spatial coordinate system in S. Therefore Eqn(42) must hold for all values of u; in particular, it holds when $u = 0$, $\gamma_u = 1$ and $x' \to x''$, giving

$$x''(\mathrm{T2}) + L1'' = x''(\mathrm{T2}) + L1 \tag{43}$$

so that

$$L1'' = L1 \tag{44}$$

A similar argument for the frames S and S', considering the configuration of P and T1 at time $\tau = \tau1 - L1/v$, shows that:

$$L1' = L1 \tag{45}$$

so that Eqn(19) is verified.

The formulae (2)-(4), as derived from the geometry of Figs.1-3, are correct. However, if the configurations of Figs.2 and 3 are to correctly represent (as claimed by Sartori) observations in the frames S' and S", respectively, of the coincidence events defined in the frame S in Fig.1, then the PVAR of Eqn(1) must be replaced by the Relative Velocity Addition Relation (RVAR) Eqn(5.19) of Ref. [5], which corresponds, in the notation for velocities in Figs.1-3, to:

$$w = \gamma_v(u - v) \quad \text{(Fig.2)} \tag{46}$$

$$w = \gamma_u(u - v) \quad \text{(Fig.3)} \tag{47}$$

while the speed v of P in Fig.2 is replaced by $v\gamma_v$ and u in Fig.3 by $u\gamma_u$ (see Figs. 3 and 4 of Ref. [5]). Thus also v in Eqn(3) should be replaced by $v\gamma_v$, u in Eqn(4) by $u\gamma_u$. The velocity w in Eqn(3) is replaced by that in Eqn(46), that in Eqn(4) is by that in Eqn(47). The times and separations in Eqns(5)-(10) are then replaced by those presented in the first row of Table 2 in Ref. [5]. It is found that $L2 = L2' = L2''$ and $L3 = L3' = L3''$ so that there is no special 'length contraction' effect as claimed in Eqns(20) and (21).

Also, in contradiction to what is shown in Figs.4 and 5, events at times connected by the TD relation ($\Delta\tau 21' = \gamma\Delta\tau 21$ in Fig.4, $\Delta\tau 31 = \gamma\Delta\tau 31'$ in Fig.5) correspond to the same spatial coincidence events (P-T2 in the primary experiment in Fig.4, T1-T2 in the reciprocal experiment in Fig.5). The distinction between $\Delta\tilde{\tau}21$ and $\Delta\tau 21$ and between $\Delta\tilde{\tau}31'$ and $\Delta\tau 31'$ claimed in these figures and the accompanying calculations is therefore illusory.

What are actually shown in Figs.2 and 3 —configurations reciprocal to those shown in Fig.1 and related to them by the PVAR of Eqn(1)— are configuration of related but *physically independent* space-time experiments in which the initial velocity parameters u and v of Fig.1 are replaced by v and w in Fig.2 and by u and w in Fig.3. In the nomenclature introduced in Refs. [6, 5] what are shown in Figs.2 and 3 are 'base frame' configurations in S' and S" that are reciprocal to the one in S shown in Fig.1. Physically distinct TD effects (see Eqns(7.1)-(7.3) of Ref. [5]) occur in the three different experiments.

The claim that Sartori's equations (26) and (27) above are erroneous because incompatible with Eqns(30) and (31) is wrong since, as explained above, these equations (taken from Eqns(20) and (21) respectively) are incorrect. However, the essential critique of Sartori's derivation of the PVAR —the irrelevance, given the existence of synchronised clocks at different, and in principle, arbitary, positions in an inertial frame of a physical local clock in specifying the proper time interval in the TD relation, and the use of TD relations in an experiment and its reciprocal (which are physically independent) as though they related observations of the same events in different frames in the primary experiment— remains valid. In particular, the demonstration of the self-contradictory nature, as shown in Eqn(34), of any attempt to associate the TD effect of the reciprocal experiment with observations of the same events in different frames in the primary experiment is not affected by any of the previous calculational or conceptual errors in the paper. Sartori's 'derivation' of the PVAR therefore remains fortuitous.

References

[1] L.Sartori,'Elementary derivation of the ralativistic velocity addition law', Am. J. Phys. **63** 81-82 (1995).

[2] A.Einstein,'Zur Elektrodynamik bewegter Korper', Annalen der Physik **17**, 891-921 (1905). English translation by W.Perrett and G.B.Jeffery in 'The Principle of Relativity' (Dover, New York, 1952) P37-P65, or in 'Einstein's Miraculous Year' (Princeton University Press, Princeton, New Jersey, 1998) P123-P161.

[3] R.Mansouri and R.U.Sexl, 'A Test Theory of Special Relativity: I Simultaneity and Clock Synchronisation', Gen. Rel. Grav. **8**, 497-513 (1977).

[4] J.H.Field, 'A New Kinematical Derivation of the Lorentz Transformation and the Particle Description of Light', Helv. Phys. Acta. **70** 542-564 (1997); arXiv pre-print: http://xxx.lanl.gov/abs/physics/0410262. Cited 27 Oct 2004.

[5] J.H.Field, 'Primary and reciprocal space-time experiments, relativistic reciprocity relations and Einstein's train-embankment thought experiment', arXiv pre-print: http://xxx.lanl.gov/abs/0807.0158. Cited Jul 2008.

[6] J.H.Field, 'The physics of space and time III: Classification of space-time experiments and the twin paradox', arXiv pre-print: http://xxx.lanl.gov/abs/0806.3671. Cited 23 Jun 2008.